

Roll No.....

Total No. of Questions—11]

[Total No. of Printed Pages—4

11T1KJ8

1517

MATHEMATICS

PAPER—First Term

Maximum Marks—50

Time Allowed—2½ Hours

Note : In case of failures/re-appear cases and fresh private candidates; i.e. candidates appearing for the first time after having passed Secondary School Examination, marks secured out of 50 shall be raised proportionately as if obtained out of 75.

1. (a) If A, B, C are three sets, then show that :

$$A \cap (B - C) = (A \cap B) - (A \cap C).$$

(b) If $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 2x + 3$, find $f[f(x)]$.

Or

(a) Let $f(x) = 4x - 1$ for all $x \in \mathbb{R}$. Show that f is a bijection. Find $f^{-1}(-1)$. 3

(b) In a group of 65 people 40 like Cricket, 10 like both Cricket and Tennis. How many like Tennis only and not cricket? How many like Tennis? 4

2. (a) Solve the inequation :

$$\frac{3(x-2)}{5} \geq \frac{5(2-x)}{3}$$

P. T. O.

(2)

(b) If α, β be the roots of the equation $x^2 - mx + x = 0$ form the equation whose roots are

$$\left(\frac{\alpha}{\beta} + 1\right) \text{ and } \left(\frac{\beta}{\alpha} + 1\right).$$

Or

(a) Solve graphically, $x + 3y \geq 3$, $3x - 2y \leq 6$, $-x + 3y \leq 6$. 3

(b) The sum of a number and its positive square root is $\frac{3}{4}$. Find the number. 4

3. (a) If $A + B + C = \pi$, prove that :

$$\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C.$$

(b) If $2\cos\theta = x + \frac{1}{x}$, prove that :

$$2\cos 3\theta = x^3 + \frac{1}{x^3}.$$

Or

(a) Simplify :

$$\tan 25^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 65^\circ.$$

3

(b) In any ΔABC , prove that :

$$\tan \frac{B}{2} \tan \frac{C}{2} = \frac{b+c-a}{b+c+a}.$$

4

4. (a) Prove that in any triangle ABC

$$\frac{a \sin(B-C)}{b^2 - c^2} = \frac{b \sin(C-A)}{c^2 - a^2} = \frac{c \sin(A-B)}{a^2 - b^2}.$$

(b) Show that :

$$\frac{2 \cos 2\theta + 1}{2 \cos 2\theta - 1} = \tan\left(\frac{\pi}{3} + \theta\right) \times \tan\left(\frac{\pi}{3} - \theta\right).$$

Or(a) If $\tan \theta = 2 \tan \phi$, show that :

$$\frac{\sin(\theta + \phi)}{\sin(\theta - \phi)} = 3.$$

3

(b) If $\sin \theta + \operatorname{cosec} \theta = 2$, evaluate $\sin^{100} \theta + \operatorname{cosec}^{100} \theta$.

4

5. Prove by the principle of Mathematical induction

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

3

6. Solve for x

$$\log(2x + 1) - \log(2x - 1) = 1.$$

3

7. Show that :

$$(1 - w + w^2)^7 + (1 + w - w^2)^7 = 128.$$

3

8. Show that :

$$\frac{\cos 15^\circ - \sin 15^\circ}{\cos 15^\circ + \sin 15^\circ} = \frac{1}{\sqrt{3}}.$$

3

9. Find two numbers whose A.M. is 25 and G.M. is 20.

3

10. (a) If a, A, b are in A.P., then A =

1

(b) The sum to infinity of a G.P. (when $|r| < 1$), a, ar, ar^2, \dots is

1

(c) Given $\log 2 = 0.3010$, then find $\log 80$.

1

(d) Find x, if $2x - 3 > 5$.

1

11. Choose the correct answer from the alternatives given against each of the following items :

(i) The sum of the roots of the quadratic $x^2 - 5x + 6 = 0$ is

- (a) - 5
- (b) 5
- (c) 6
- (d) None of these.

1

(ii) The modulus of the complex number $3 - 4i$ is

- (a) - 5
- (b) 5
- (c) 1
- (d) 4.

1

(iii) For any three sets A, B, C $A \times (B - C)$ equals :

- (a) $A \times B - A \times C$
- (b) $- A \times B + A \times C$
- (c) $A + (B - C)$
- (d) None of the above.

1