FEDERAL BOARD OF INTERMEDIATE AND SECONDARY EDUCATION ISLAMABAD

MATHEMATICS HSSC–I

SECTION – A (Marks 20)

Time allowed: 25 Minutes

Version Number 3 1 1 3

Note: Section – A is compulsory. All parts of this section are to be answered on the separately provided OMR Answer Sheet which should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q. 1 Choose the correct answer A / B / C / D by filling the relevant bubble for each question on the OMR Answer Sheet according to the instructions given there. Each part carries one mark.

1) What is the range of $y=\sin^{-1}x$?

A. $\frac{-\pi}{4}<y<\frac{\pi}{2}$ B. $0<y<\pi$ C. $\frac{-\pi}{2}\le y\le\frac{\pi}{2}$ D. $0\le y\le\pi$

2) What is the general solution of $\sin x=0$ in $\mathbb{R}$?

A. $\left\{\pm\frac{n\pi}{2}:n\in\mathbb{Z}\right\}$ B. $\left\{\pm\frac{3n\pi}{2}:n\in\mathbb{Z}\right\}$

C. $\{\pm n\pi:n\in\mathbb{Z}\}$ D. $\{\pm 2n\pi:n\in\mathbb{Z}\}$

3) Under which of the following operations, the set $S=\{-1,0,1\}$ is closed?

A. Multiplication B. Division C. Addition D. Subtraction

4) Which of the following sets is equal to $\{x\in\mathbb{Q}:x^2=2\}$

A. $\{\ \}$ B. $\mathbb{Q}$ C. $\{\pm\sqrt{2}\}$ D. $\{\pm 1\}$

5) Which of the following binary relations from $A=\{1,2,3\}$ to $B=\{a,b,c\}$ is a function?

A. $\{(1,a),(2,c),(2,b)\}$ B. $\{(1,a),(2,b),(1,c)\}$

C. $\{(1,a),(1,b),(2,c),(3,c)\}$ D. $\{(1,a),(2,a),(3,c)\}$

6) Let A and B be the square matrices of the same order. Which of the following is true about A and B?

A. $\det(A)=\det(B)$ B. $\det(AB)=\det((AB)')$

C. $\det(A+B)=\det A+\det B$ D. $\det(AB)=\det(BA)$

7) If two roots of a cubic equation are 0 and i, then the cubic equation is:

A. $x^3-x=0$ B. $x^3-1=0$ C. $x^3+1=0$ D. $x^3+x=0$

8) What could be the partial fractions of $\frac{x^2+2x+4}{(x-2)(x^3-8)}$?

A. $\frac{A}{x+2}+\frac{B}{(x-2)^2}+\frac{C}{x^2-2x+4}$ B. $\frac{A}{x+2}+\frac{B}{(x-2)^2}+\frac{Cx+D}{x^2+2x+4}$

C. $\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{Cx+D}{x^2-2x+4}$ D. $\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{Cx+D}{x^2+2x+4}$

9) What is the sum of n terms of the sequence with n^{th} term $a_n=4n+1$?

A. $2n(2n+3)$ B. $n(2n+3)$ C. $2n+3$ D. $4n+6$

10) What is the sum of the series $1+\frac{1}{3}+\frac{1}{9}+\ldots\ldots\ldots$?

A. $\frac{3}{4}$ B. $\frac{3}{2}$ C. 3 D. $\frac{4}{3}$

11) If a fair die is rolled, then what is the probability that the top is a prime number?

A. $\frac{2}{5}$ B. $\frac{3}{2}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$

12) For what values of x, the binomial expansion of $\left(2-\frac{x}{2}\right)^{-1}$ is valid?

A. $|x|>4$ B. $|x|>2$ C. $|x|<4$ D. $|x|<2$

13) How many lines can be drawn between the five points in a plane?

A. 120 B. 60 C. 20 D. 10

14) Which term is the middle term in the expansion of $\left(x-\frac{2}{x}\right)^{2n}$?

A. $(n-1)^{th}$ term B. $\left(\frac{n}{2}-1\right)^{th}$ term C. $\left(\frac{n}{2}+1\right)^{th}$ term D. $(n+1)^{th}$ term

15) The radian measurement of the central angle of a circle of radius $6cm$ which cuts off an arc of $12cm$ long is:

A. 3 B. 4 C. 1 D. 2

16) Which of the following indentities is **TRUE**?

A. $\sin 3\theta = 3\sin\theta + 4\sin^3\theta$ B. $\sin 3\theta = 4\sin\theta + 3\sin^3\theta$

C. $\cos 3\theta = 4\cos^3\theta + 3\cos\theta$ D. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

17) Which of the following is equal to $\cos\left(\frac{3\pi}{2}-x\right)$?

A. $\sin x$ B. $\cos x$ C. $-\cos x$ D. $-\sin x$

18) What is primary period of $\frac{1}{2}\sin 2x$?

A. 2π B. $\frac{\pi}{2}$ C. 4π D. π

19) In a right angle triangle ABC, if the lengths of two non-perpendicular sides are 5 and 3, then what will be the length of the third side?

A. 4 B. $\sqrt{34}$ C. 3 D. 4.5

20) If R is circumradius of a triangle ABC, Then $R=$

A. $\frac{abc}{4\Delta}$ B. $\frac{4\Delta}{abc}$ C. $\frac{abc}{\Delta}$ D. $\frac{abc}{4}$

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FEDERAL BOARD OF INTERMEDIATE AND SECONDARY EDUCATION ISLAMABAD

MATHEMATICS HSSC–I

Time allowed: 2:35 Hours **Total Marks Sections B and C: 80**

NOTE: Attempt any ten parts from Section 'B' and any five questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Graph paper will be provided on request.

SECTION – B (Marks 40)

Q. 2 Attempt any TEN parts. All parts carry equal marks. **(10 x 4 = 40)**

(i) Express the complex number $1+i\sqrt{3}$ in polar form.

(ii) Show that $(A\cup B)'=A'\cap B'$ (Demorgan's Law). Where A and B are subsets of a universal set U.

(iii) If a,b are elements of a group G under the operation of multiplication. Then show that $(ab)^{-1}=b^{-1}a^{-1}$

(iv) If $A=\left[a_{ij}\right]_{3\times 3}$, and $\lambda\in\mathbb{R}$, then show that $\lambda A-A=(\lambda-1)A$

(v) Determine whether $p\to(q\to p)$ is a tautology, a contingency or an absurdity.

(vi) Discuss the nature of roots of $2x^2-5x+1=0$.

(vii) If a number exceeds its square root by 56. Find the number.

(viii) Find the $13th$ term of the sequence $x, 1, \; 2-x, \; 3-2x, \ldots$

(ix) Find the sum of n terms of the series whose n^{th} term is $3n^2+n+1$.

(x) A box contains 10 red, 30 white and 20 black marbles. A marble is drawn at random. Find the probability that it is either red or white.

(xi) If x is so small that its square and higher powers can be neglected, then show that $\dfrac{\sqrt{4+x}}{(1-x)^3}\cong 2+\dfrac{25}{4}x$

(xii) Prove that $\dfrac{\sin^2(\pi+\theta)\tan(\frac{3\pi}{2}+\theta)}{\cot^2(\frac{3\pi}{2}-\theta)\cos^2(\pi-\theta)\cos ec(2\pi-\theta)}=\cos\theta$

(xiii) If a triangle ABC is with $a=\sqrt{3}-1$, $b=\sqrt{3}+1$ and $\gamma=60^\circ$ then find c .

(xiv) Without using calculator or table, prove that $2\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{7}=\dfrac{\pi}{4}$

SECTION – C (Marks 40)

Note: Attempt any FIVE questions. All questions carry equal marks. **(5 x 8 = 40)**

Q. 3 Find the value of λ for which the system:

$$\begin{aligned}x+y+z&=0\\2x+y-\lambda z&=0\\x+2y-2z&=0\end{aligned}$$

has a non-trivial solution. Also solve the system.

Q. 4 Show that roots of $x^2+(mx+c)^2=a^2$ will be equal if $c^2=a^2(1+m^2)$

Q. 5 Sum the following series to n terms: $\dfrac{1^2}{1}+\dfrac{1^2+2^2}{2}+\dfrac{1^2+2^2+3^2}{3}+\ldots$ to n terms.

Q. 6 By the principle of mathematical induction, show that $x+y$ is a factor of $x^{2n-1}+y^{2n-1}$ $(x\neq -y)$, for all positive integer n.

Q. 7 Without using calculator / table, prove that $\sin 10^\circ\sin 30^\circ\sin 50^\circ\sin 70^\circ=\dfrac{1}{16}$

Q. 8 In a triangle ABC, with usual notations, prove that:

Area of Triangle $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$ (The Hero's Formula)

Q. 9 Solve the trigonometric equation $\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta=0$ for its general solution.

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MATHEMATICS HSSC–I

SECTION – A (Marks 20)

Time allowed: 25 Minutes **Version Number | 3 | 1 | 1 | 8**

Note: Section – A is compulsory. All parts of this section are to be answered on the separately provided OMR Answer Sheet which should be completed in the first 25 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q. 1 Choose the correct answer A / B / C / D by filling the relevant bubble for each question on the OMR Answer Sheet according to the instructions given there. Each part carries one mark.

1) The circumradius of an equilateral triangle with length of a side $4m$ and area Δ in square meters is:

A. $\frac{64}{\Delta}$ B. $\frac{4}{\Delta}$ C. $\frac{8}{\Delta}$ D. $\frac{16}{\Delta}$

2) If $\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}-x$, then find x

A. $\frac{\pi}{3}$ B. $\frac{\pi}{4}$ C. $\frac{2\pi}{3}$ D. $\frac{\pi}{6}$

3) What is the general solution of the trigonometric equation $\cos x=1$?

A. $\{2n\pi : n\in\mathbb{Z}\}$ B. $\left\{\frac{\pi}{2}+2n\pi : n\in\mathbb{Z}\right\}$

C. $\left\{\frac{n\pi}{2} : n\in\mathbb{Z}\right\}$ D. $\left\{\frac{2\pi}{3}+2n\pi : n\in\mathbb{Z}\right\}$

4) Which of the following numbers is **rational**?

A. e^2 B. $\sqrt{\frac{3}{2}}$ C. π D. $\sqrt{\frac{12}{75}}$

5) What is the modulus of the complex number $\frac{3-4i}{4+3i}$?

A. 1 B. 5 C. -1 D. $\frac{1}{5}$

6) Which of the following sets is equal to the set $\{n\in\mathbb{Z} \mid n^2=-n\}$

A. $\{-1,-2,-3,-4,...\}$ B. $\{0,-1\}$

C. ϕ D. $\{-1,-4,-9,-16,...\}$

7) The set of natural numbers under the operation of addition is:

A. Monoid B. Group

C. Abelian group D. Not a semigroup

8) Let $AX=O$ be a system of linear equations in matrix form, with rank (A) less than the number of unknowns. Then system has:

A. A non-trivial solution B. More than one finite many solutions

C. Infinite many solutions D. A trivial solution only

9) Find the value of $\begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 2 & -1 & -2 & 0 \\ -4 & 1 & 3 & 4 \end{vmatrix}$

A. 24 B. –24 C. 36 D. –36

10) Let $f(x)$ be a polynomial of degree three such that $f(1) = 0 = f(2) = f(0)$ then $f(x) = \ldots\ldots.$

A. $x^3 - 3x^2 - 2x$ B. $x^3 + 3x^2 + 2x$ C. $x^3 - 3x^2 + 2x$ D. $x^3 + 3x^2 - 2x$

11) What are the distinct roots of the equation $x^2 + x^{-2} - 2 = 0$?

A. $\pm 1, \pm i$ B. $\pm i$ C. ± 2 D. ± 1

12) Which of the following types **may be** the partial fractions of the expression $\dfrac{x^2}{(1-x^2)(4+x)}$?

A. $\dfrac{A}{1-x^2} + \dfrac{Bx+C}{4+x}$ B. $\dfrac{A}{1-x} + \dfrac{B}{1+x} + \dfrac{C}{4+x}$

C. $\dfrac{Ax}{1-x^2} + \dfrac{Bx+C}{4+x}$ D. $\dfrac{Ax+B}{1-x^2} + \dfrac{C}{4-x}$

13) If A.M and G.M between a and b are equal, then $(a-b)^2 =$

A. $2ab$ B. $\sqrt{ab}$ C. 0 D. $4ab$

14) If $a_1 = -1,\ a_n = n + a_{n-1}$, then find the sum S_3 of first three terms.

A. 5 B. 6 C. 3 D. 4

15) In how many ways a cricket team of 11 players can be selected out of 15 players if the captain must be included in each way?

A. $15!$ B. $\overset{14}{\underset{4}{C}}$ C. $\overset{15}{\underset{11}{C}}$ D. $11!$

16) A coin is tossed twice. What is the probability that all two will be the same?

A. $\dfrac{3}{4}$ B. $\dfrac{1}{2}$ C. 1 D. $\dfrac{1}{4}$

17) What is the sum of the series $\binom{7}{0} + \binom{7}{2} + \binom{7}{4} + \binom{7}{6}$?

A. 128 B. 64 C. 48 D. 32

18) The radian measure of the central angle of a sector of the circle is $40°$ and radius of the circle is $3m$. What is the area of the sector?

A. $2\pi\ m^2$ B. $\pi\ m^2$ C. $3\pi\ m^2$ D. $4\pi\ m^2$

19) What is the range of $y = \tan x$?

A. $0 < y < \infty$ B. $\mathbb{R}$ C. $-1 \le y \le 1$ D. $-\infty < y < 0$

20) For what value of k, the primary period of $\dfrac{1}{3}\cos kx$ is $\dfrac{2\pi}{3}$?

A. 9 B. 3 C. 6 D. $\dfrac{3}{2}$

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MATHEMATICS HSSC–I

Time allowed: 2:35 Hours **Total Marks Sections B and C: 80**

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SECTION – B (Marks 40)

Q. 2 Attempt any TEN parts. All parts carry equal marks. **(10 x 4 = 40)**

(i) Simplify the following by justifying each step: $\dfrac{4+16x}{4}$

(ii) Simplify the following complex number by expressing in the form $a+ib$

$$\frac{2}{\sqrt{5}+\sqrt{-8}}$$

(iii) Find all the fourth roots of unity.

(iv) Complete the following table, to obtain that $S=\{a,b,c\}$ is a semigroup under the operation $*$.

$*$	a	b	c
a	c	a	b
b	a	b	c
c	…	…	a

(v) Find matrix X if $\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$

(vi) Find the numerical value of k if the polynomial x^3+kx^2-7x+6 has a remainder -4 when divided by $x+2$.

(vii) Resolve $\dfrac{3x-11}{(x^2+1)(x+3)}$ into partial fractions.

(viii) If $y=1+\dfrac{x}{2}+\dfrac{x^2}{4}+\ldots$, then show that $x=2\left(\dfrac{y-1}{y}\right)$

(ix) Find n if ${}^{n}P_{4} : {}^{n-1}P_{3} = 9:1$

(x) Evaluate $\sqrt[3]{32}$ correct to three places of decimal, by using binomial expansion.

(xi) Prove that $(\sec\theta-\tan\theta)^2=\dfrac{1-\sin\theta}{1+\sin\theta}$

(xii) Prove that $\tan 56^\circ=\dfrac{\cos 11^\circ+\sin 11^\circ}{\cos 11^\circ-\sin 11^\circ}$ without using calculator.

(xiii) A ladder leaning against a vertical wall makes an angle of 24° with the wall. Its foot is $5m$ from the wall. Find its length.

(xiv) Without using table / calculator, prove that $\sin^{-1}\dfrac{1}{\sqrt{5}}+\cot^{-1}3=\dfrac{\pi}{4}$

SECTION – C (Marks 40)

Note: **Attempt any FIVE questions. All questions carry equal marks.** **(5 x 8 = 40)**

Q. 3 Prove that $\sim q \wedge (p\to q) \to \sim p$ is a tautology, where p and q are any two logical statements.

Q. 4 Solve the following system by reducing their augmented matrix to the Echelon form:

$$\begin{aligned} x_1+4x_2+2x_3&=2 \\ 2x_1+x_2-2x_3&=9 \\ 3x_1+2x_2-2x_3&=12 \end{aligned}$$

Q. 5 Prove that $\dfrac{x^2}{a^2}+\dfrac{(mx+c)^2}{b^2}=1$ will have equal roots, if $c^2=a^2m^2+b^2$; $a\neq 0, b\neq 0$

Q. 6 Find the n^{th} term of the geometric sequence if $\dfrac{a_5}{a_3}=\dfrac{4}{9}$ and $a_2=\dfrac{4}{9}$.

Q. 7 Find the 6^{th} term in the expansion of $\left(x^2-\dfrac{3}{2x}\right)^{10}$.

Q. 8 Prove the fundamental law of trigonometry: $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$, where α,β are two real angles.

Q. 9 For any triangle ABC with usual notations show that: $r_1=4R\sin\dfrac{\alpha}{2}\cos\dfrac{\beta}{2}\cos\dfrac{\gamma}{2}$